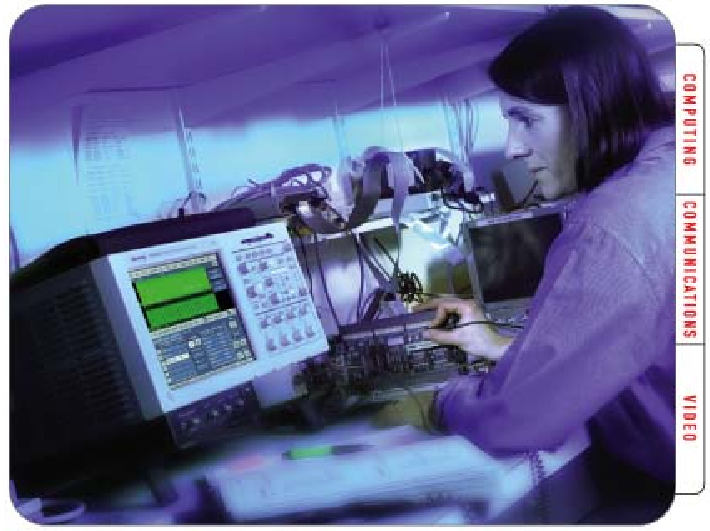
Bandwidth and the Influence of Digital Signal Processing



Introduction

The most universally recognized figure of merit for any oscilloscope is its bandwidth - the frequency at which the magnitude of the frequency response is reduced by 3 db relative to the magnitude at DC. Digital storage oscilloscopes (DSOs) use advanced digital signal processing (DSP) to enhance accuracy and performance, but not all DSOs are created equal. Some DSOs apply DSP to extend their bandwidth – a practice that, if unknown to the user, can yield unexpected waveform displays and faulty analysis.

In the past, a DSO's advertised bandwidth was typically the true analog bandwidth of the scope. However, with some recent introductions to the high-end oscilloscope market, it is becoming essential to question what this advertised specification really means. This brief looks beyond advertised (and over simplified) DSO bandwidth claims based on the use of DSP to reveal the actual frequency response that may be much lower than the true analog bandwidth required to support reliable, accurate measurements. Side-by-side tests illustrate the potential tradeoffs resulting from use of DSP in a DSO to extend bandwidth.

TRUE ANALOG BANDWIDTH IS CONSTANT

Figure 1 illustrates the three basic elements of a DSO input system – probe/amplifier, analog-to-digital converter (ADC), and DeMUX/memory. The probe/amplifier and the ADC determine the true analog bandwidth of the DSO. These circuits operate at a constant, continuous rate regardless of sample rate, number of channels or trigger conditions.

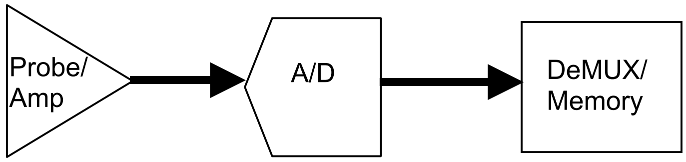


Figure 1. *This block diagram shows three blocks: Probe and amplifier, ADC, DeMUX/memory*.

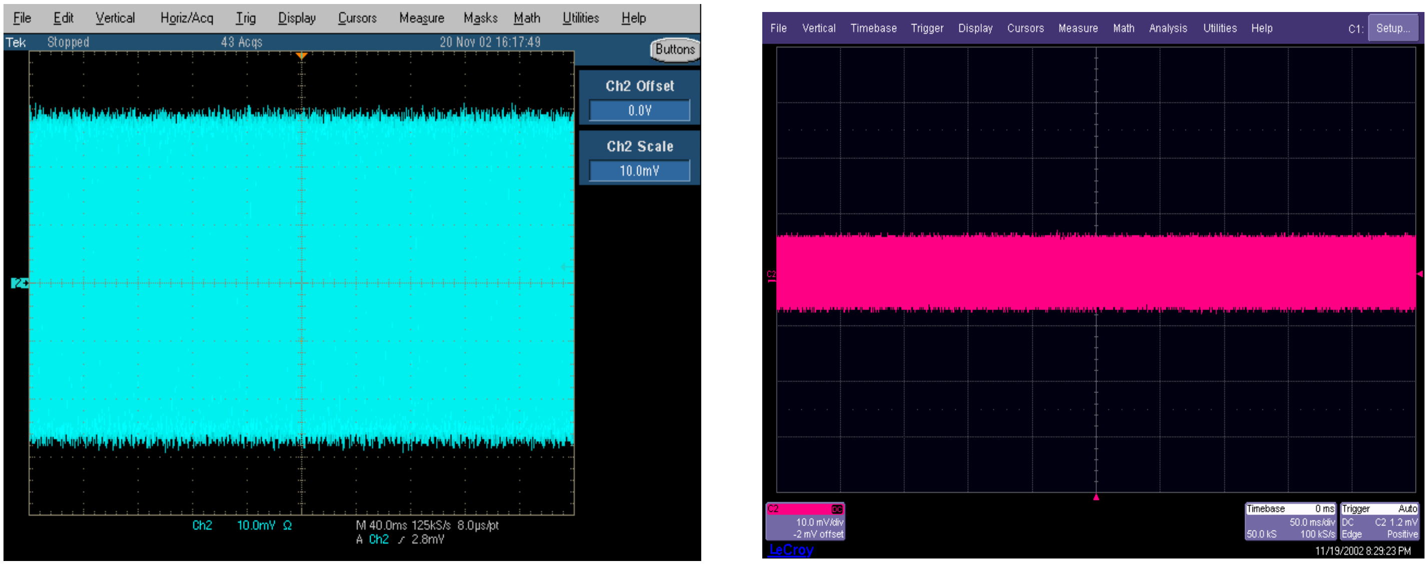
A LOW-BANDWIDTH DSO WITH DSP IS STILL A LOW-BANDWIDTH DSO

As a rule of thumb, we need to question whether the full advertised bandwidth of a DSP-enhanced DSO is available, regardless of how signals are acquired, sampled or interpolated. Unless a DSO can retain its entire advertised analog bandwidth when the DSP is removed, its performance will be compromised under practical measurement conditions.

Here are some specific clues to detect bandwidth limitations. At first glance, the Tektronix TDS6604 and LeCroy WM8600A may appear to be equivalent products. The TDS6604 is a high-performance DSO with 6GHz analog bandwidth at the probe tip. The WM8600A advertises "6GHz Analog Bandwidth," but closer inspection reveals some significant differences in how each company achieves their advertised bandwidths.

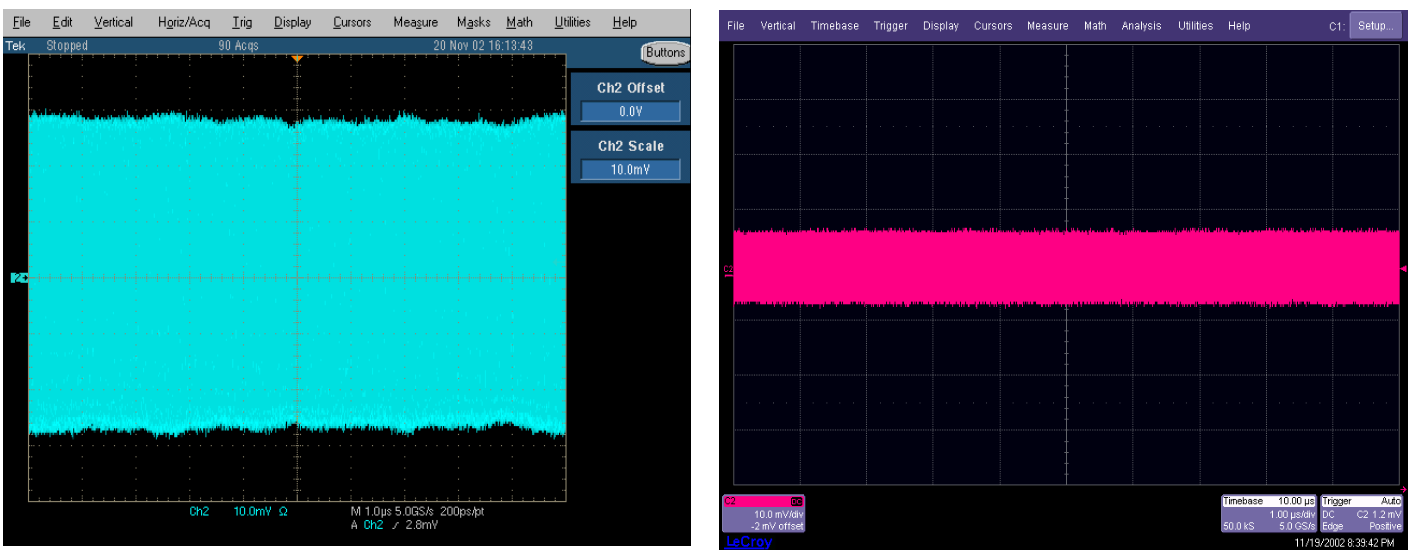
#1: Magnitude changes at lower sample rates

Every DSO operates at multiple sample rates to accommodate a wide variety of applications and measurements. Sampling rates below the maximum are accomplished through a decimation process before the data enters memory (not by changing the clock frequency). The magnitude of the true analog frequency response is not changed by decimation, but post-processing DSP can change the frequency response. Figures 2, 3 and 4 compare the frequency response of the two DSO at three different sample rates. The TDS6604's magnitude of frequency response remains constant with sample rate - post processes do not introduce high frequency content beyond that which was digitized. The TDS6604 displays and measures the data exactly as it was digitized. High frequency performance of the WM8600A is degraded at less than its maximum sample rate.



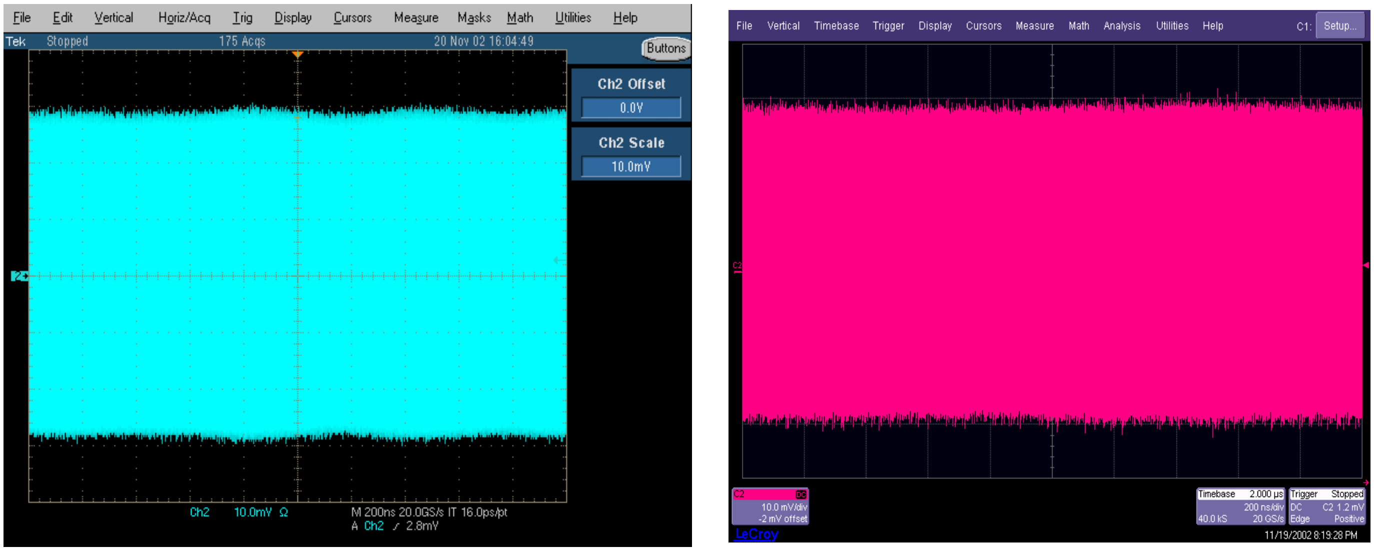
Tektronix TDS6604 LeCroy WM8600A

Figure 2. *The magnitude of the frequency response at 5.9 GHz sampled at 125 kS/s and 100 kS/s respectively.*



Tektronix TDS6604 LeCroy WM8600A

Figure 3. *Magnitude of the frequency response at 5.9 GHz sampled at 5 GS/s.*



Tektronix TDS6604 *LeCroy WM8600A*

Figure 4. *Magnitude of the frequency response at 5.9 GHz sampled at 20 GS/s*

The ability to measure R.F. envelopes or glitches using peak detect is compromised by low bandwidth.

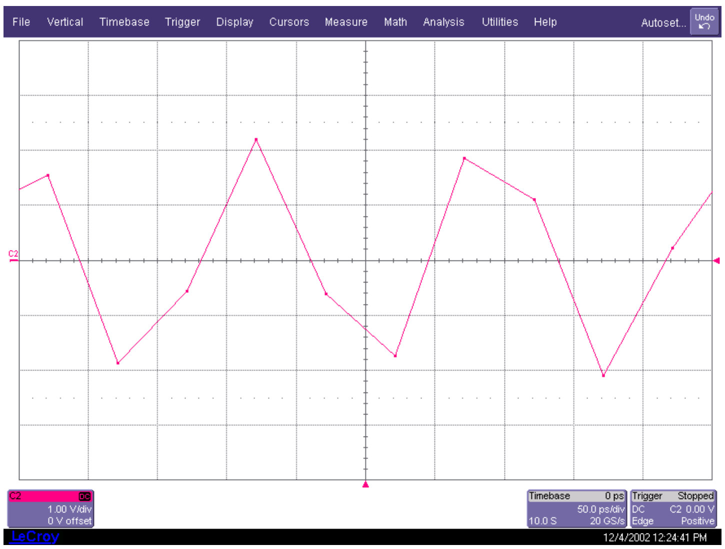
#2 Limited real-time sampling displays

Analog bandwidth, defined as the –3 dB point, represents the highest frequency signal a DSO can accept without adding distortion. Real-time digital bandwidth (sometimes called single-shot bandwidth) defines the maximum frequency the DSO can acquire by sampling the entire input waveform in one acquisition, using a single trigger, and still gather enough samples to reconstruct the waveform accurately.

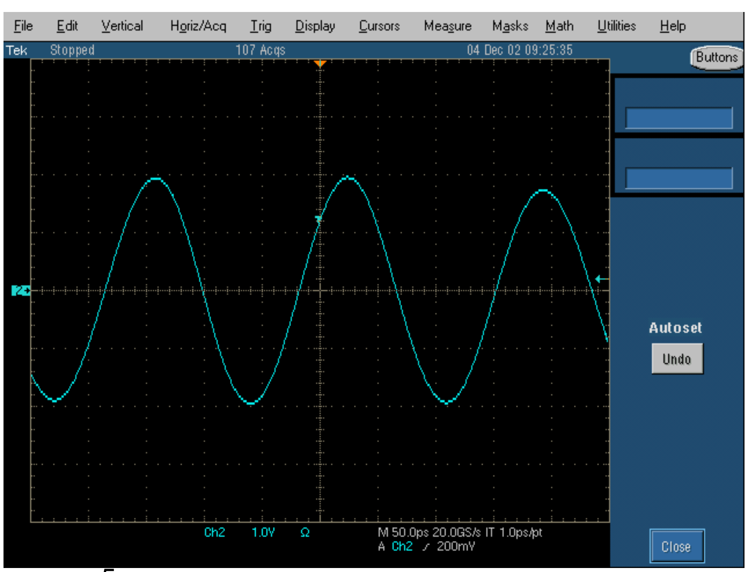
Real time sampling acquires all samples from a waveform in a single triggered acquisition - the waveform need not be repetitive to be properly reconstructed. In the fastest real time oscilloscopes currently available, the waveform is sampled every 50 ps (20 GS/s rate). According to Nyquist's sampling theorem for a limited bandwidth (band-limited) signal with maximum frequency fmax, the equally spaced sampling frequency fs must be more than twice the maximum frequency f_{max} ($f_s > 2 \cdot f_{max}$) in order to reconstruct the signal without aliasing. In order to reconstruct the original signal from the sampled signal, an oscilloscope interpolates between the data points – i.e., it draws lines between the samples on the display, creating a virtually continuous waveform instead of a string of individual points.

However, as Figure 5 shows, the use of linear interpolation for reconstruction of the waveform data between sample points can lead to a grossly inaccurate reconstruction. In this example, a 6 GHz sine wave was sampled every 50 ps (20 GS/s) and reconstructed using linear interpolation. This is clearly not adequate even though 6 GHz is well below the Nyquist frequency (10 GHz). Sin(x)/x interpolation is thus needed to display the acquired signal more accurately (and is the type of interpolation implied by the Nyquist sampling theorem).

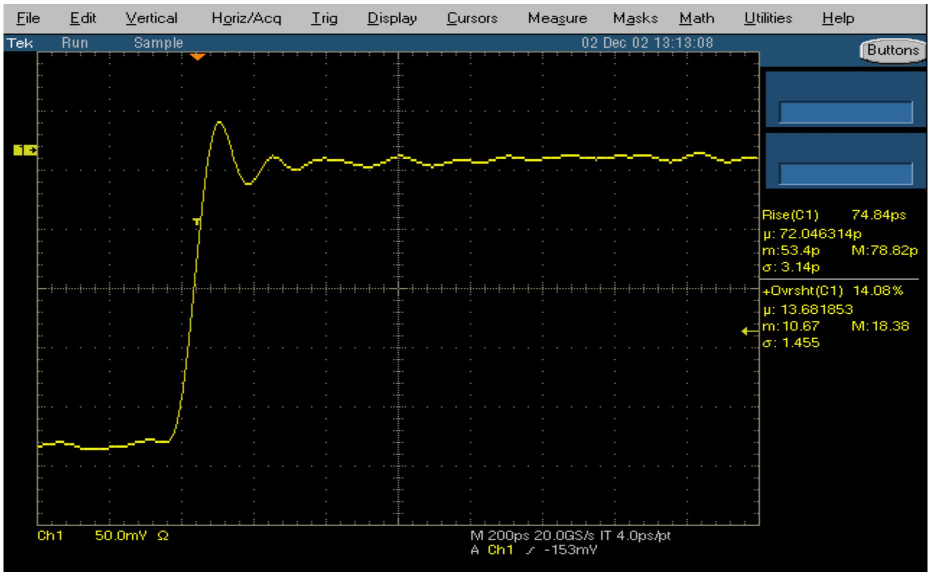
Figure 7 shows how Sin(x)/x interpolation produces high-resolution timing and amplitude measurements as well as accurate displays.



LeCroy WM8600A
Figure 5. *Without Sin(x)/x interpolation the information is limited. Linear interpolation is the default mode. Sin(x)/x interpolation is available on the LeCroy instrument, not as the default mode, but as a menu selection.*



TDS6604
Figure 6. *Data is Sin(x)/x interpolated by a factor of 50 from 20 GS/s, giving a point every 1 ps. Sin(x)/x is the default mode.*

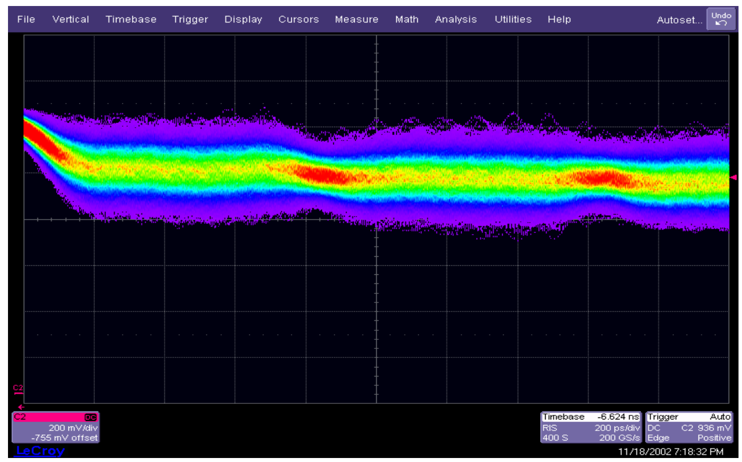


TDS6604
Figure 7. *Waveform Sin(x)/x interpolated by a factor of 12.5 from 20 GS/s to 4 ps/pt reproducing a fast step with excellent fidelity*

As figures 6 & 7 show, the Sin(x)/x curve interpolation algorithms are optimized for a wide variety of waveforms. Tektronix instruments execute Sin(x)/x DSP quickly, providing excellent throughput. For these reasons, Sin(x)/x is the default mode on Tektronix oscilloscopes, although the DSP may be turned off by the user if desired. While not as readily available as the default, Sin(x)/x is accessible on LeCroy instruments. The Appendix presents a more complete description of the Sin(x)/x interpolation used in the TDS5000, 6000, and 7000 Series.

#3: Magnitude and bandwidth change at higher sample rates

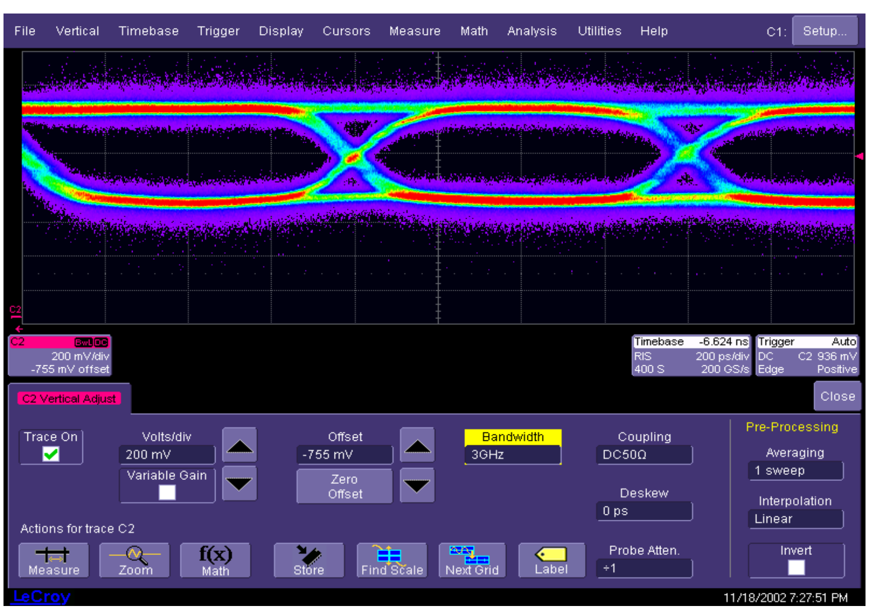
Random Equivalent Time sampling, (sometimes called "Repetitive" or R.I.S) is available for precision sampling of ***repetitive*** signals beyond the maximum real time sampling rates of a DSO (currently 20 GS/s). For RET to provide this increased timing resolution, the signal must be repetitive and the scope must be triggered at the same point on each successive waveform. When the first trigger event occurs, a 20 GS/s DSO captures a set of points spaced 50 ps apart. On the next trigger event, the DSO captures another set of points spaced 50 ps apart, but slightly offset in time from the first acquisition. This process continues until the scope has acquired the desired number of sample points. These sample points, acquired over multiple acquisitions, are then interleaved together to construct a Random Equivalent Time record with resolution much finer than that available in a single shot without interpolation. Due to the fact that sample points are acquired over multiple trigger events, no DSP or interpolation is performed on RET data. Just as in real time sampling though, the magnitude of the frequency response is still determined by the probe/amp and ADC of the DSO (remember – true analog bandwidth of your oscilloscope never changes).



TDS6604 – 6 GHz bandwidth *WM8600A – "Full" bandwidth*

Figure 8. *Serial data waveforms at 1.25 Gb/sec RET sampled at 5 ps intervals, the equivalent of 200 GS/s.*

Figure 8 shows the same 1.25 Gb/sec serial data acquired by both the TDS6604 and WM8600A in RET, which LeCroy calls RIS. Notice that the WaveMaster 8600's amplitude is much lower and the waveshape is incorrect. DSP bandwidth boost creates these errors by attempting to artificially increase the bandwidth (virtual bandwidth).



WM8600A

Figure 9. *Operating at 3 GHz, (with no DSP bandwidth extension), the low frequency Amplitude is believable, but the display is quite noisy.*

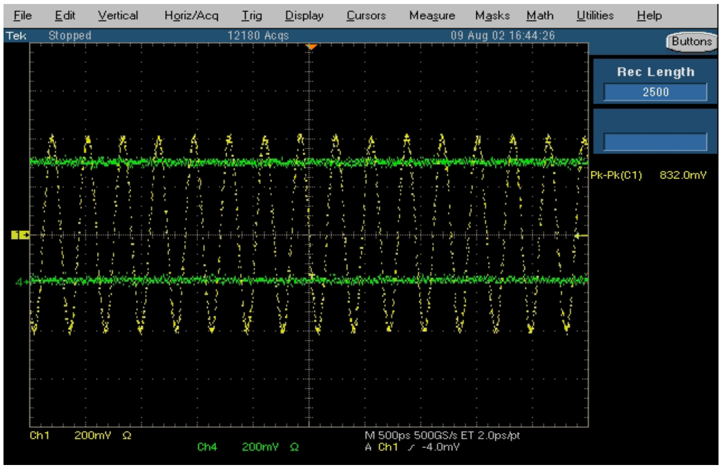
In Figure 9, the bandwidth extension DSP is turned off. With only 3 GHz bandwidth, the bandwidth extension errors are not present.

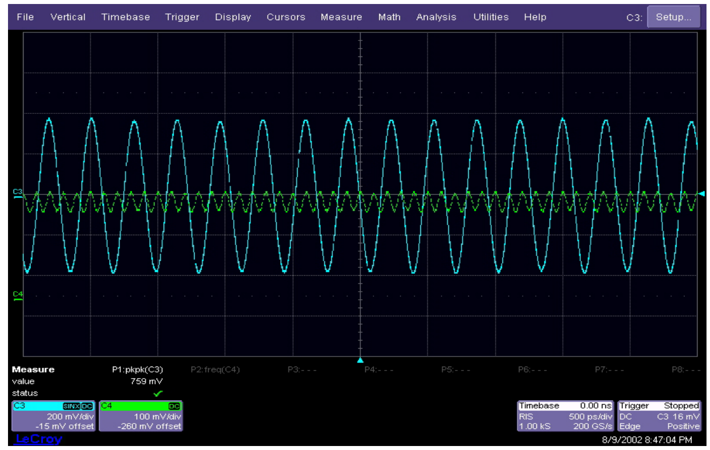
#4: Bandwidth changes with selection of trigger source.

As stated several times in this document, true analog bandwidth is defined by the front-end components of the oscilloscope (probe/amp and ADC). This true analog bandwidth never changes; it will always be there. Virtual bandwidth created by DSP, however, changes quite often and with little or no notification to the user. This could easily lead to incorrect measurement results even when the scope operator appears to be doing everything correctly.

An example of this is trigger source (the channel being used to trigger the scope). In this case, the DSP algorithm gives differences in amplitude (bandwidth) depending on which trigger source is selected, as illustrated in the following figures.

Figure 10 compares amplitude vs. trigger source results on the two DSOs. Both instruments have a sine wave applied to channel one and a square wave applied to channel 2, and both instruments are triggered on channel one. The amplitude of both signals is displayed correctly on the TDS6604. The amplitude of the square wave is inaccurately represented on the WM8500 scope on the right.

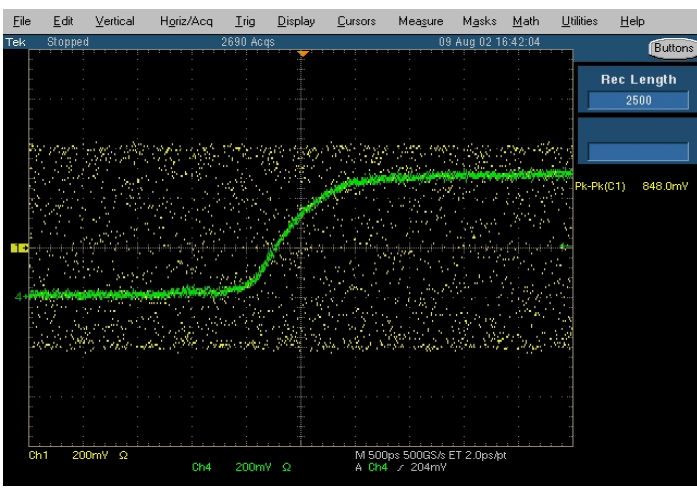


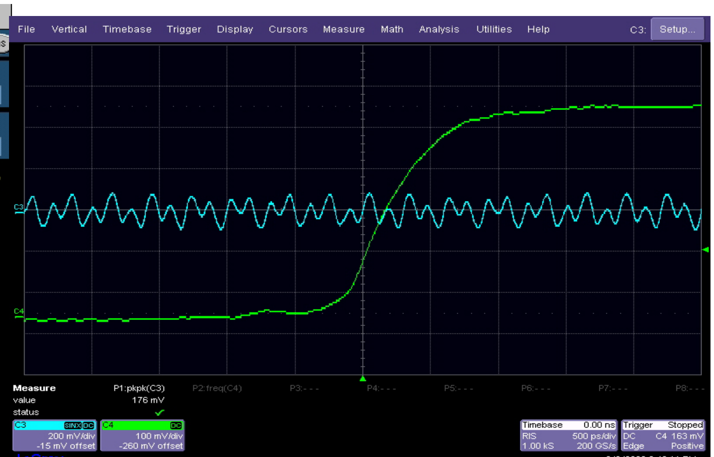


TDS6604 *LeCroy WM8500*

Figure 10. *Sine and square wave test – triggered on channel 1.*

In Figure 11, both instruments once again have a sine wave applied to channel one, and a square wave applied to channel two. The only difference is in this case both instruments are triggered on the square wave on channel two. Notice how the TDS6604 accurately displays the amplitude of the sine wave, just as it did when the sine wave was the trigger source. The WM8500 displays a much lower amplitude sine wave (bandwidth) when the instrument is triggered on the square wave. The Tektronix TDS6604 demonstrates advertised analog bandwidth regardless of trigger source; the WM8500 does not.





TDS6604 *LeCroy WM8500*

Figure 11. *Sine and square wave test – triggered on channel 2.*

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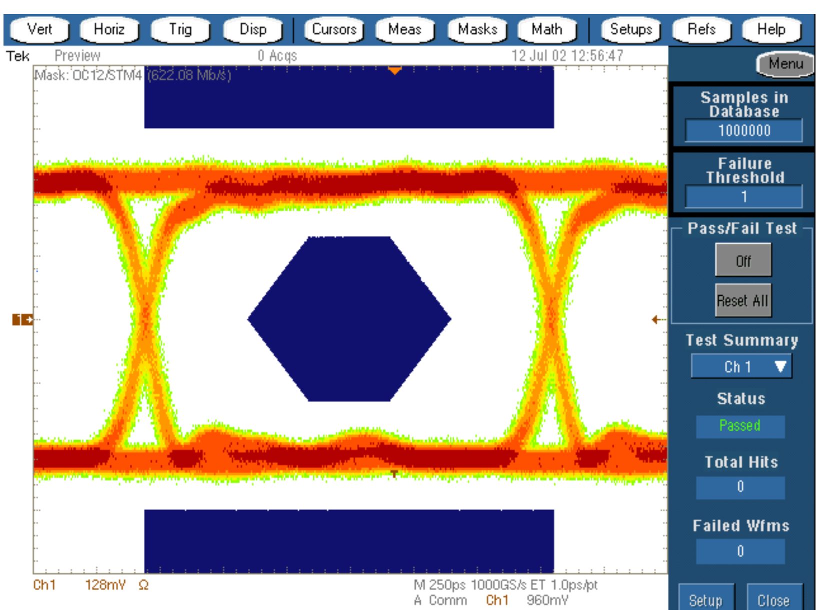
Examples of appropriate use of DSP in a DSO

DIGITAL FILTERING

In cases where the application is specific or the environment and the expected results are predictable, DSP has proven useful and even essential. Digital filtering has proven to be a valuable asset in the calibration of the optical magnitude and phase to a very specific industry standard for critical compliance measurements. For this specialized test, DSP is used to change the bandwidth of the oscilloscope to comply with the OC-48 Optical Reference Receiver definition. For compliance, the optical magnitude must be within 0.5 db of the industry standard Bessel-Thompson fourth order response. A DSP filter is used for this specialized optical compliance test (Figures 12 and 13).

The flexibility and accuracy of digital filtering permit a single instrument to cover more standards than would be practical with switched hardware filters. The digital filter can be turned off when the DSO returns to general-purpose use beyond the specialized testing modes.

Tektronix CSA7404
Figure 12. *Optical Reference Receiver frequency response for OC-48.*



Tektronix CSA7404
Figure 13. *Optical Mask Testing using Optical Reference Receiver.*

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Conclusion

Modern performance oscilloscopes need to produce reliable results in a variety of applications. DSP has proven useful and even essential for accurate interpolation near the Nyquist frequency as well as serial data compliance measurements.

The true analog bandwidth of a DSO is determined by the performance of the front-end components (probe/amp and ADC), and does not change as various scope settings or parameters are adjusted. If the scope is relying on DSP to increase the bandwidth to achieve the advertised bandwidth, be positive that you understand ***every*** scenario or setup where this practice could lead to unexpected measurement results.

How Sin(x)/x Interpolation Works

The TDS Series scopes expand waveforms by using a digital signal processing technique that reduces the sample rate requirements for sine waves to about 2.2 per cycle. This method of interpolation produces higher resolution timing and amplitude measurements than linear interpolation, as well as more accurate displays. The following discussion explains the technique, which is essentially a linear filtering process.

When the TDS oscilloscope acquires a continuous-time input signal $\hat{X}(t)$ at a sampling rate with period T, it saves the acquisition as a sequence of equally spaced samples:

$$x(n) = \hat{X}(nT), n = 0, 1, 2, \ldots, N$$

where *N* is the selected record length.

(Note that $\hat{X}(t)$ must be bandlimited to a frequency $f_m < \frac{1}{2T}$ to prevent aliasing.)

With approximations of *t* and *n* extending to ±∞, the Fourier transforms of $\hat{X}(t)$ and *x(n)* are $\hat{X}(\omega)$ and $X(e^{j\omega T})$, respectively.

Figure 7 shows the relationship between $|\hat{X}(\omega)|$ and $|X(e^{j\omega T})|$ when $T = \frac{1}{2fm}$.

Note that:

$$\left|X\left(e^{j\omega T}\right)\right| = \frac{1}{T}\left[\left[\hat{X}(\omega)\right]\right], -\frac{\pi}{T} \leq \omega \leq \frac{\pi}{T}.$$

The goal of interpolation is to create a sequence that would result from sampling the input signal at a faster rate. If the sampling rate is increased by an integer factor *L*, the sampling period is decreased to $T' = \frac{T}{L}$. Therefore the desired sequence is:

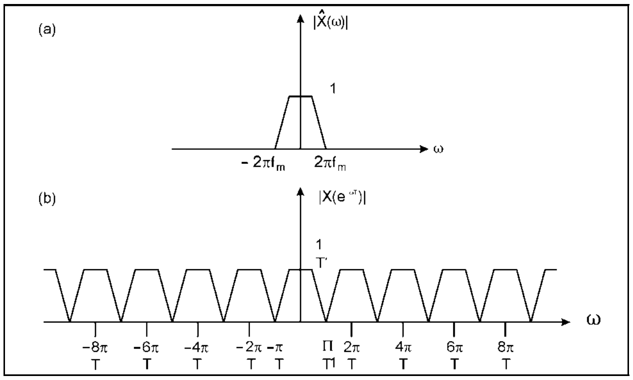

Figure 7. This illustration shows: **a.** The magnitude Fourier transform of the input signal $\hat{X}(t)$, **b.** The magnitude Fourier transform of the saved-waveform sequence *x*(n).

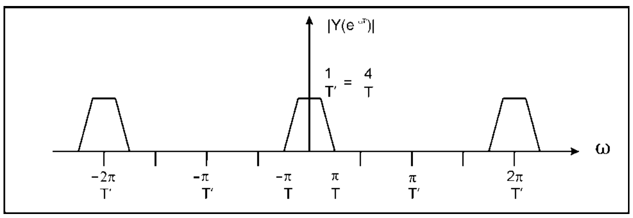

Figure 8. This illustration shows the magnitude Fourier transform of the sequence y(n) with L=4. This sequence is the one that would be obtained by sampling the input signal $\hat{X}(t)$ four times faster.

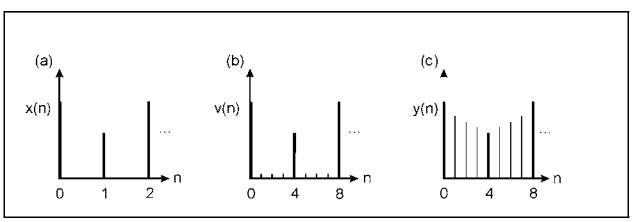

Figure 9. The TDS5000, 6000, 7000 Series expands waveforms in two steps, as illustrated here for L=4. The intermediate sequence v(n) results from effectively sampling the acquired sequence x(n) four times faster. When properly filtered, v(n) produces the desired sequence y(n).

$y(n) = \hat{x}(nT')$.

The Fourier transform of *y(n)* must be:

$$Y\left(e^{j\omega T'}\right) = \frac{1}{T'}\left[\hat{X}(\omega)\right], -\frac{\pi}{T'} \leq \omega \leq \frac{\pi}{T'}.$$

Figure 8 shows $|Y(e^{j\omega T'})|$ for the case where $L = 4$. Comparing Figure 8 to Figure 7b suggests the use of a filter to create the higher sample-rate sequence by adjusting the gain of $X(e^{j\omega T})$ and removing the unwanted images centered at:

$$\pm\frac{2\pi}{T}, \pm\frac{4\pi}{T}, \pm\frac{6\pi}{T}, \pm\frac{10\pi}{T}, \pm\frac{12\pi}{T} \ldots$$

This is just what the TDS Series scopes do in a two-step process. Refer to Figure 9.

The first step toward obtaining the desired sequence *y(n)* is, in effect, to sample the saved-waveform sequence *x(n)* at the higher sampling rate, which is achieved by inserting L–1 zero-valued points after each acquired sample in *x(n).* The resulting sequence is:

$$v(n) = x\left(\frac{n}{L}\right), = 0, L, 2L, \ldots$$

=0 otherwise

The sequence *v(n)* has sampling period T′.

The Fourier transform of the new sequence *v(n)* turns out to be identical to the Fourier transform of the sequence shown in Figure 7b, that is:

$$V(e^{j\omega T'}) = X(e^{j\omega T})$$

However, the Nyquist frequency has been increased by the factor *L*, which makes it

possible to obtain *y(n)* by applying a suitable digital low-pass filter to *v(n).*

Ideally this filter must:

- be periodic with period $\frac{2\pi}{T'}$,
- have a gain of *L,* and
- reject frequencies between $\frac{\pi}{T}$ and $\frac{\pi}{T'}$.

Such a filter has Fourier transform of

$$H\left(e^{j\omega T'}\right) = L, -\frac{\pi}{T} \leq \omega \leq \frac{\pi}{T}$$
$$= 0, \frac{\pi}{T} < |\omega| < \frac{\pi}{T'}$$

The TDS Series waveform interpolators approximate this ideal filter with a Finite-duration Impulse Response (FIR) digital filter. This extremely efficient filter delivers high throughput for displaying and measuring expanded waveforms. Furthermore, the FIR filter accurately passes frequency components up to 90% of the Nyquist frequency. The resulting useful storage bandwidth (USB) is:

$$USB(MHz) = \frac{SamplingRate(MHz)}{2.2}$$

The TDS Series interpolator filter is also quite powerful for pulse-type waveforms. The filter provides nearly perfect expansion as long as the rise and fall time of the pulse time is greater than 1.5 times the sampling period.